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FUZZY DECISION-MAKING IN WATER RESOURCE PROBLEMS

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ABSTRACT

Many problems in Water Resources involve Multiobjective decision-making (MODM) in which the decision-maker faces the task of selecting the best course of action so as to meet, in the best manner possible, several varied objectives which may be non-commensurable, mutually conflicting or difficult to quantify. Apart from these difficulties, cases also arise where the objectives and/or constraints are 'fuzzy', namely inexact or uncertain.

Fuzzy decision-making, which is based on Fuzzy Set theory, enables the decision-maker to select the best policy in a fuzzy environment. Levels of satisfaction μ_i are defined as ramp functions in the interval (0,1) for each of the fuzzy goals (objectives) and fuzzy constraints. The figure of merit for each policy is reckoned as Min μ_i, and the optimal policy is that which maximizes the overall level of satisfaction, i.e., Max Min μ_i. Objectives that are qualitative can be assigned appropriate values for μ. The importance of Fuzzy DM in the analysis of water resource problems is emphasized in the paper.

INTRODUCTION

Many Water Resources problems are concerned with 'decision-making' to select the best policy or course of action. Examples are :

* Reservoir operation to optimize objectives of electric power generation, irrigation, flood control, water supply etc.
* Conjunctive exploitation of surface water and ground water to meet irrigation or water supply demands, subject to water quality and other constraints.
* Integrated basin development, involving several reservoirs, to meet a variety of objectives.
* Drought control and management.

These problems usually involve several objectives of varying importance. Examples of such objectives are:

Category	Objective	Scale
Economic	Project cost	Monetary
	Power production	Monetary
	Irrigation	Monetary or areal
	Flood Protection	Monetary or cumec

Category	Objective	Scale
Water quantity & quality	Water yield	M m^3
	Low flow augmentation	cumec
	Concentration of dissolved/ suspended matter	mg/lit
Environmental	Sedimentation	m^3/km^2
	Neighbourhood improvement	Qualitative
	Wildlife preservation	Qualitative
	Aesthetics	Qualitative
Recreational	Tourism revenue	Monetary

Selection of the optimal policy in such problems involves Multiobjective decision-making (MODM), an important branch of Operations Research.

STATEMENT OF THE MODM PROBLEM

The problem can be stated as

$$\text{Maximize } f_i(x), \quad i = 1, 2, \ldots I$$

subject to the constraints

$$g_j(x) \geq 0, \; j = 1, 2, \ldots J; \quad x \geq 0$$

where the objective functions f(x) and the constraints g(x) are defined on a n-dimensional space of decision variables

$$x = (x_1, x_2, \ldots, x_n)$$

The 'feasible region' of the decision variables is determined solely by the constraints, and is the set

$$X = \{x: \text{such that } g_j(x) \geq 0, x \geq 0\}$$

It should be noted that the statement 'Maximize $f_i(x)$' is not unambiguous, since one cannot maximize a vector of objective functions without some basic assumptions. For the same reason, the term 'optimization' is also ambiguous when applied to MODM problems.

Neumann and Morgenstern have pointed out that multiobjective decision-making is not a maximization problem, and not even a classical mathematical problem. It involves resolution of conflict in a situation where simultaneous attainment of all the objectives at desired levels may not be possible.

Solution of MODM problems is generally carried out in two stages: (i) Finding the set of 'efficient solutions' either partially or in full; and (ii) Choosing the 'best alternative' from the efficient solutions, based on some criteria for 'aggregating' the different objectives.

The set S of efficient solutions, which is a subset of the set X of feasible solutions, is defined as follows:

S = $\{x^* : x^* \in X$, subject to the condition that there is no $x \in X$ such that for some $i = q$, $f_i(x) > f_i(x^*)$ and for all other i, $f_i(x) \geq f_i(x^*)\}$

This means that as we move from one efficient solution to another, one or more objective function improves but, at the same time, one or more of the remaining objective functions decreases in value. Efficient solutions are also referred to as 'nondominated' or 'pareto-optimal' solutions.

Generating the set of efficient solutions is relatively straightforward, since each objective can be maximized independently of the others to yield a possible efficient solution.

THE CRUX OF MODM

The crux of MODM problems lies in weighting the objectives appropriately and in choosing the best alternative from among the identified set of efficient solutions. Goicoecha et al[1] and Szidarovszky et al[2] have classified the different techniques of solving MODM problems, and have discussed the methods of solution. These methods can be roughly grouped into three categories:

A. Methods that are used only to find efficient solutions: 1. Weighting method; 2. ϵ-Constraint method; 3. Multiobjective Simplex method of Philip; 4. Multiobjective Simplex method of Zeleny.

In the weighting method, the decision-maker assigns relative weights to the objective functions which are then simply added to yield a single objective problem. The weights can be varied to yield a set of nondominated solutions. The ϵ-constraint method selects one primary objective function f_k for maximization, relegating all other objectives to the constraint set with assigned minimum levels ϵ_i. For linear problems, the simplex methods are convenient to apply. Even if the problem is one of maximization, the objective functions can be maximized and minimized to yield the 'best' and 'worst' nondominated solutions.

B. Methods that are used to find the optimal solution, based on prior allocation of weights: 5. Goal programming; 6. Compromise programming.

Goal programming involves minimizing the sum of weighted 'distances' from goals g_i set by the decision-maker for each objective.

$$Min \sum_{i=1}^{I} w_i \cdot |g_i - f_i|$$

The decision-maker has two parameters, w_i & f_i, at his control for each objective. In Compromise programming, a vector based on weighted distances from the ideal solutions f_i^* is minimized:

$$Min \sum_{i=1}^{I} w_i^p \cdot \left(\frac{f_i^* - f_i}{f_i^* - f_i^{**}} \right)^p$$

where f_i^* and f_i^{**} are the best and worst computed values for f_i, obtained by single-objective optimization, and p is a 'power parameter' in the range $1 \leq p < \infty$, which reflects the decision-maker's concern for restricting deviations from f_i^*.

C. Methods that employ interactive weighting of objectives: 7. ELECTRE I method; 8. ELECTRE II method; 9. Surrogate Worth Trade-off method; 10. SEMOPS method; 11. STEM method. Discussion of these methods is beyond the scope of this paper.

FUZZY DECISION-MAKING

The special characteristics of MODM problems in Water Resources are: (i) Diversity of objectives; (ii) Varying importance of objectives; (iii) Non-commensurable nature of some of the objectives; and (iv) Difficulty of assigning numerical scales to some of the objectives.

Water resource decision-making problems get further complicated by the fact that the objective functions and constraints are often fuzzy, i.e. inexact or 'non-crisp'. Examples of such goals and constraints are: 1. The annual revenue from hydropower production should be at least Rs.10 million ($\mu = 0$) and preferably more than Rs.15 million ($\mu = 1$); 2. The area irrigated should be not less that 15,000 ha ($\mu = 0$) and should preferably be 20,000 ha or more ($\mu = 1$); 3. The effect of the project on wildlife should not be poor ($\mu = 0$), but at least fair ($\mu = 0.4$) and preferably excellent ($\mu = 1$). In the above statements, μ represents the level of satisfaction or level of attainment of the goal or constraint.

Conventional methods cannot handle fuzzy objectives and constraints without making the solution messy. On the other hand, Fuzzy DM techniques can handle inexact or even vague goals and constraints in a mathematically elegant manner. Before taking up Fuzzy DM for a detailed discussion, it is desirable to be aware of its mathematical foundation, namely Fuzzy Set theory.

FUZZY SETS

The theory of Fuzzy Sets, first presented by Zadeh[3] in 1965, paved the way for interesting developments in many branches of science and engineering such as Operations Research, Computer Software, Process Control, etc. Fuzzy Single-objective and Multiobjective decision-making has emerged as a promising field of application of Fuzzy Sets.

A fuzzy set A in X is a set of ordered pairs:

$$A = \{x, \mu_A(x)\}, x \in X$$

where μ_A is the membership function in the interval (0,1) representing the grade of membership of x in A. For example, X may be a group of 6 persons P, Q, . . U, and the fuzzy set A = {Tall persons} in which $\mu_A(P) = 0$, $\mu_A(Q) = \mu_A(R) = 0.3$, $\mu_A(S) = 0.5$, $\mu_A(T) = 0.8$, and $\mu_A(U) = 1$, based on an assumed variation of the membership function μ with 'height of person'. In the above example, P is too short to be a member of the fuzzy set A, whereas U is tall enough to merit full membership.

Conventional set operations such as union, intersection, sum, difference, product, complement, equivalence, implication and absolute difference can be extended to fuzzy sets.

Examples: The union of two fuzzy sets $C = A \cup B$ is defined by $\mu_C(x) = \max(\mu_A(x); \mu_B(x))$. This is written as

$$\mu_{A \cup B} = \mu_A \vee \mu_B$$

Similarly, the intersection of two fuzzy sets $C = A \cap B$ is defined by $\mu_C(x) = \min(\mu_A(x); \mu_B(x))$, and is written as

$$\mu_{A \cap B} = \mu_A \wedge \mu_B$$

THE FUZZY LP PROBLEM

The basic approach of fuzzy decision-making is best explained with reference to a fuzzy linear programming (FLP) problem. The FLP problem is characterized by fuzzy goals and constrainTs which, interestingly, can be treated alike.

These are expressed in the form of a matrix

$$\sum_j a_{ij}\, x_j \leq b_i$$

Each value of b_i is associated with a level of satisfaction μ_i, which is expressed as a function of b_i. The simplest form of μ_i is the ramp function: Linear variation from 0 to 1 in a specified region of b, and no variation on either side of that region.

Example : The cost of a project, which is a linear function of three decision variables, should preferably be $\leq b_1$, but may go upto a maximum of $b_1 + d_1$. The fuzzy goal can then be stated as $a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \leq (b_1 + d_1) - \mu_1 d_1$, where μ_1 is the level of satisfaction of the goal 1.

With $i = 1, 2, \ldots I$ goals and constraints, the overall level of satisfaction for a set of values of the decision variables $(x_1, x_2, \ldots x_n)$ is taken to be Min μ_i. Since the objective of the FLP problem is to maximize μ, we attempt Max Min μ_i. The FLP problem therefore reduces to a conventional LP problem of maximizing $\mu(0,1)$ subject to the constraints.

Example : The irrigation water release from a reservoir can be utilized for either paddy or maize cultivation. The irrigation canal commands 10,000 hectares of cultivable land. In the interest of crop rotation, it is desirable to limit the cultivation in a season to 7,000 ha, though more areA can be cultivated if necessary. The water input for paddy and maize cultivation are 1.2 ha.m/ha and 0.6 ha.m/ha respectively. In order to conserve reservoir storage, the project authorities wish to limit the seasonal release to 6,000 ha.m, though they are willing to release upto 8,400 ha.m if essential. The seasonal returns from paddy and maize are Rs.12,000/ha and Rs.8,000/ha respectively. The farmers, as a group, expect a minimum seasonal return of Rs. 84 million, though they will be happy if it is not less than Rs.96 million. Work out the decision strategy.

Solution : Let x_1 and x_2 represent the area in 1,000 ha under paddy and maize cultivation respectively. Let 1 monetary unit be taken as Rs.4,000/-.

Goal (returns)	:	$3x_1 + 2x_2 \geq 21 + 3\mu$	(i = 1)
Land constraint	:	$x_1 + x_2 \leq 10 - 3\mu$	(i = 2)
Water constraint	:	$1.2x_1 + 0.6x_2 \leq 8.4 - 2.4\mu$	(i = 3)

The FLP problem can be stated as:

$$\begin{aligned} \text{Max} \quad & \mu\ (0,1), \text{ subject to} \\ & \mu \leq -7.000 + x_1 + 0.667x_2 \\ & \mu \leq 3.333 - 0.333x_1 - 0.333x_2 \\ & \mu \leq 3.500 - 0.500x_1 - 0.250x_2 \\ & x_1, x_2 \geq 0 \end{aligned}$$

This reduces to the ordinary LP problem:

Max $z = \mu + 0.\ x_1 + 0.\ x_2$, subject to the above constraints. The solution obtained by the Simplex method is:

$x_1 = \underline{3.7}$ (1000 ha); $x_2 = \underline{5.4}$ (1000 ha);
Max Min $\mu = \underline{0.300}$.

It is worth noting that single-objective and multiobjective FLP problems can be solved with the same ease by the above approach. Non-commensurable objectives pose no problem since all goals and constraints are expressed in terms of the parameter μ and the decision variables. The method can also handle fuzzy coefficients in the goals and constraints, i.e. coefficients stated as fuzzy numbers. An example of such a coefficient is (14, 12, 16): This is a triangular fuzzy number having $\mu = 1$, 0 and 0 at 14, 12 and 16 respectively. Rules are available for arithmetical operations involving fuzzy members.

The limitations of this approach should also be taken note of: (i) This method is applicable only when it is possible to place realistic bounds on the fuzzy goals and constraints. (ii) Implied in this method is the assumption that all the goals and constraints have equal weight. In some MODM problems this may not be true; for example, hydropower production and tourism revenue may deserve different weights. (iii) Inability to handle qualitative objectives which cannot be expressed as a function of the decision variables; for example, the attainment of wild life preservation goal can be described only by linguistic variables which can then be assigned numerical ratings in a standard interval of (0,1) or (0,100).

THE FUZZY MODM PROBLEM

The transformation of a fuzzy MODM problem to a fuzzy LP problem, indicated in the previous section, may not always be feasible. In its most general form, a fuzzy MODM problem may feature:

1. Fuzzy goals; 2. Fuzzy constraints; 3. Fuzzy coefficients in the equations describing the goals and/or constraints; 4. Subjective ratings in the form of crisp or fuzzy numbers for some of the objectives; this may be done even for quantifiable objectives that are considered to be not important enough to merit detailed analysis; 5. Linguistic variables to describe some of the objectives that are vague, e.g. aesthetics; these variables are replaced by crisp or fuzzy numbers at a later stage in the analysis.

In large MODM problems, the feasible approach is to arrive at a set of efficient solutions and then subject these to a critical analysis for selecting the best policy. The efficient solutions, referred to as 'alternatives', are evaluated with respect to a set of 'criteria' represented by the goals (Table 1). For example, C_1 may be hydropower, C_2 may be irrigation release, C_3 may be tourism revenue, and so on.

Table 1

		ALTERNATIVES				
		A_1	A_2	A_3	-	A_m
C	C_1	-	-	-		-
R	C_2	-	-	-		-
I	C_3	-	-	-		-
T	-					-
E	-					-
R	-					-
I						
A	C_n	-	-	-	-	-

The alternatives chosen should all be nondominated solutions with respect to those goals that can be expressed as functions of the decision variables. In order to give adequate representation to those objectives that can be expressed only by subjective ratings or linguistic variables, the decision-maker should include in the set some alternatives which provide desirable levels of attainment of those goals.

Bardossy and Duckstein[4] have presented an aquifer management problem studied in Hungary, in which 6 alternative policies were evaluated for 6 objectives. Ideal and worst points were identified for each objective, and the ratings of all objectives were normalized in the interval (0,1). Weights w_i were also assigned for the 6 objectives with $\Sigma w_i = 1$. One of the objectives, aesthetics, was described by linguistic variables ranging from 'excellent' to 'poor' for the 6 alternatives. These, in turn, were represented by fuzzy numbers in the interval (0,1). Fuzzy compromise programming with $p = 6$ was carried out to select the best alternative. It was found that the selected alternative was not the best in respect of any one of the 6 objectives.

The basic philosophy of Fuzzy DM is one of 'satisficing' the goals in the best manner possible. This is in contrast to the concept of vector minimization of 'distances from an ideal point' followed in many of the Non-fuzzy DM techniques. In this context, the very concept of 'level of satisfaction' of a policy assumes significance. Although in Fuzzy LP problems the level of satisfaction of goals provided by a policy is defined as Min μ_i, this definition suffers from three shortcomings: (i) A single low μ_i can make a policy worthless; (ii) Unless the μ of a particular goal is the minimum, it has no effect on the assessed worth of a policy; and (iii) Weighting of goals is not possible.

Kickert[5] has presented an interesting discussion on the above aspect which is at the core of Fuzzy DM. Other proposed alternatives such as $\mu = \Pi_i \mu_i$ and $\mu = \bar{\mu}_i$ are not demonstrably superior to $\mu = \text{Min } \mu_i$.

Kickert has also presented an alternative fuzzy ranking procedure, originally proposed by Baas and Kwakernaak, which lends itself excellently to Fuzzy MODM. In this approach, any alternative A_i receives the weighted average rating

$$\bar{R}_i = \sum_{j=1}^{n} w_j \cdot r_{ij} / \sum_{j=1}^{n} w_j$$

where r_{ij} is the merit or rating of the alternative A_i according to the criterion (goal) C_j; and w_j is the relative importance of that criterion. Both r_{ij} (which corresponds to the μ_i of Fuzzy LP) and w_j can be crisp or fuzzy numbers, or even linguistic variables (such as very good, good, fair, very important, moderately important or unimportant) that are subsequently transformed into fuzzy numbers in the interval (0, 1). In case r_{ij} and/or w_j are fuzzy, evaluation of $\overline{R}_i$ involves fuzzy arithmetic. Finally $\overline{R}_i$ induces a ranking of the alternatives $A_1, A_2, \ldots, A_m$.

There is difference of opinion among experts in the field as to which ranking procedure best suits Fuzzy MODM. The search for the ideal rule of composition of fuzzy levels of satisficing continues.

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